Simple, Fast, and Unconditionally Stable Direct Nonlinear Analysis using nathan-α Method

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Abstract: The availability of advanced high strength material and structural system requiring the use of nonlinear analysis, with materially nonlinear or geometrically nonlinear effects, especially in seismic area. The use of existing nonlinear analysis procedures are time consuming, more difficult to apply and in some methods, can not go beyond limit points. To cope with these problems, a new nonlinear analysis procedure, that is more stable, more simple and faster, called Direct Nonlinear Analysis using α Method, has been developed. The new procedure has been tested for single and multidimensional problems with good results. The method can be extended to various problems, and can be improved to make it even faster or more stable using more detail algorithm. The availability of this new method will open new field of research in nonlinear analysis to provide us with better nonlinear analysis solution.

Keywords: *Fast Nonlinear analysis, Direct Nonlinear Analysis, Alpha Method, DNA-α Method, nathan-α Method, Newton-Raphson Method, Arc-Length Method, Riks-Wempner Method, Crisfield Sphere Method, Mean Value Theory, Post-Buckling Analysis, Strain hardening, Materially Nonlinear, Geometrically Nonlinear, Incremental Iterative Nonlinear Analysis, Push Over Analysis, Second Order Analysis, Unconditionally stable.*

Introduction

Nonlinear analysis usually required for problems with very soft material with nonlinear behavior even at small forces or deformations, such as in geotechnical problems, or in problems involving very large forces (as in seismic load) capable to turn any material to behave nonlinearly because its stress-strain curve has passed the elastic point, or for structures exhibit large deformations requiring the use of higher order stiffness matrices formulation behaving nonlinearly to load changes.

Nonlinear Analysis Problems in Engineering Applications

In this paper, we will focus on nonlinear analysis problems in real engineering applications. The specific properties of engineering problems will be utilized to devise a new nonlinear analysis method suitable for the real problems. Usually in engineering problems the initial state is known (x=0, y=0), the model behaves linearly at early stage before reaching nonlinear stage, it also behaves linearly during unloading, the tangent stiffness matrix is always positive definite (determinant of the tangent stiffness matrix is real and exist) at the linear region. In the nonlinear region, structure can have several bifurcation points, and in certain cases, can have snap-through and snap-back behavior. Some materials can also show softening or hardening effects. Load can be applied in incremental step, and even dynamic load can be simulated by pseudo-static incremental load. In large deformation state, load-deformation relationship will be nonlinear, requiring geometrically nonlinear model (P-delta effects). Changing of boundary condition and support condition can also create nonlinear behavior. In engineering problems, tangent stiffness can be computed for every load step and available to be used for next state. Using these principles we will devise a new method that can solve any nonlinear problems, unconditionally stable, directly without any iteration. The latest AISC steel code recommends the use of nonlinear analysis as the standard procedure.

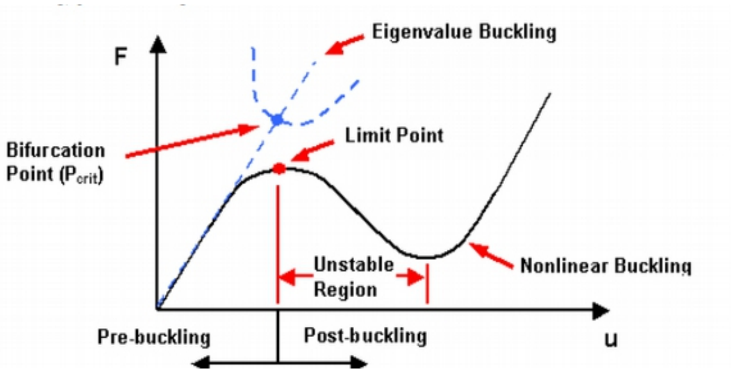


Fig. 1. Equilibrium Path with Linear and Nonlinear regions (ResearchGate.net, 2015)

Materially Nonlinear structures can be analyzed using Nonlinear material model, Elasto-plastic or Bi-linear Model, and Material Strain Hardening model.

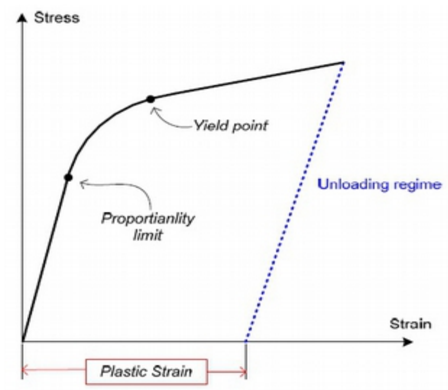


Fig.2a. Nonlinear Material (Zareh)

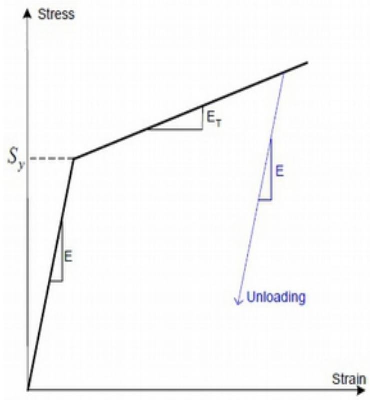


Fig.2b. Bi-linear Material

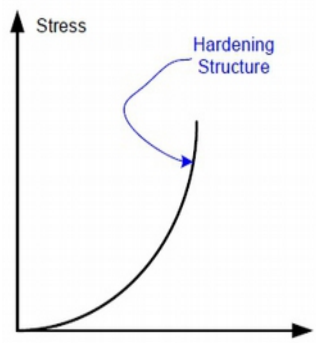


Fig.2c. Strain Hardening

Geometrically nonlinear

Certain type of structures can have buckling or snap-through or snap-back phenomena when large load causing large displacement applied.

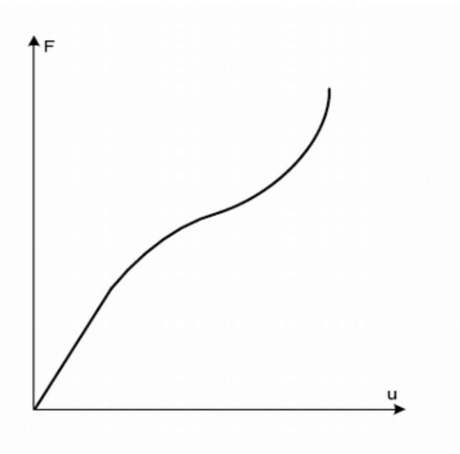


Fig. 3a. Geometrically Nonlinear (Zareh, 2008)

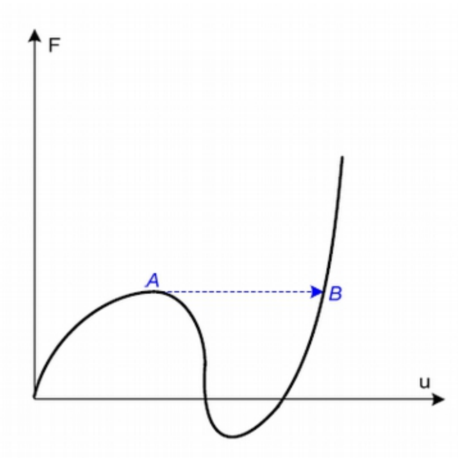


Fig.3b. Snap-through Effects (Zareh)

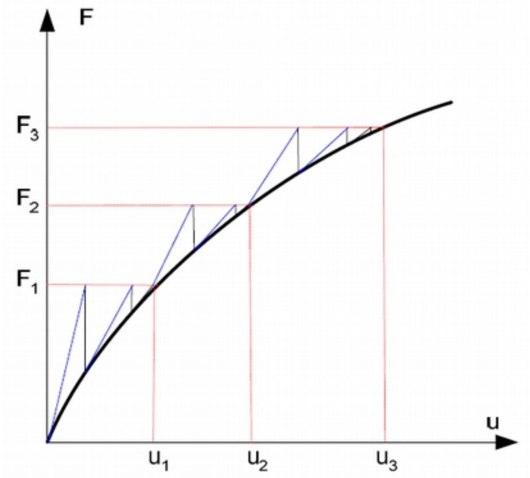
If a structural model has one or more of the nonlinearities above, the response of the structure can only be calculated accurately by an accurate and stable nonlinear analysis method.

Review of currently available Nonlinear Analysis methods

Due to its nonlinear behavior, the solution for nonlinear problems is not easy to be solved directly, but must be solved iteratively using some iteration schemes. The solution can be divided into explicit or implicit methods, where implicit method means we try to find the value of x,f(x+d) using previous value of x,f(x) and estimated value of f(x+d), while in explicit method we find the value of x,f(x+d) using only known values of x,f(x). The explicit method is straightforward, does not need iteration, but requires very small load step, while the implicit method requires more difficult calculations for iterations, but can use larger load step.

The first implicit method to solve a nonlinear equation is Newton-Raphson method (from Raphson's *Analysis Aequationum Universalis*, 1690 and Isaac Newton, 1969). Newton's method is a simple and powerful technique, with quadratic convergence behavior, but requires the function derivative (tangent stiffness matrix) evaluated in the calculation. It can diverge from the solution if the initial point is not correctly estimated, and overshoot the solution and even diverge to other solution. The method is also slow to converge for multiple roots condition or near bifurcation points.
The Modified Newton-Raphson method tries to save some valuable time by reusing the previously

calculated tangent stiffness matrix for several iterations before recalculate the tangent matrix again at converged point, by paying more time and more iterations.



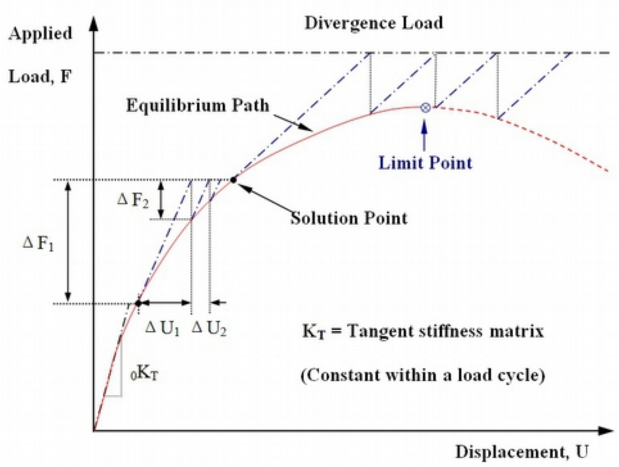


Fig.4a. The Newton-Raphson Method (1690) Fig.4b. The Modified Newton-Raphson (NIDA)

Because the Newton-Raphson method is not stable and very slow at limit points, several new methods have been invented to solve the problems: Displacement control method, Arc-Length Method (Risk, 1979 and Wempner, 1971), and Modified Arc-Length Method (Crisfield, 1981).

All methods tried to devise a way to trace next equilibrium state based on **one previous equilibrium state**. While the new proposed method will use **two previously calculated equilibrium states** to find the next equilibrium state, using **Mean Value Theory.** This is the **basic principles** of the proposed method.

From below illustrations, we can see that the current method use very complicated algorithms to trace the equilibrium path correctly, especially to determine whether the path is increasing or decreasing, and how to handle limit point conditions. For each step, there will be many calculations should be done, involving factored tangent stiffness matrix, load vectors and displacement vectors.

The Displacement Control Method

To cope with the instability of Newton-Raphson method near limit points, instead of increasing the load, the Displacement Control Method uses the displacement increment to find the next equilibrium point. In this way, the method can trace the equilibrium path passing the inflection points. The displacement control method performs satisfactorily when handling snap-through problems, but it fails at a snap-back point. Moreover, it may be very difficult, in some cases, to select a suitable displacement degree of freedom as the control parameter.

The Constant Work Method

The constant external work method to keep the work done in each load increment constant was proposed by Bathe and Dvorkin (1983). They claimed that it is more reliable than the arc-length method near a critical point. Later, Yang (1984) employed the constant work method for large deflection analysis of frames. However, Chan and Ho (1990) proved mathematically that this method is equivalent to the displacement control method with the steering displacements being the same as all the displacement degrees of freedom.

Several other analysis methods have been invented based on The Displacement Control method, The Arc-Length Method or mixed of the two approaches.

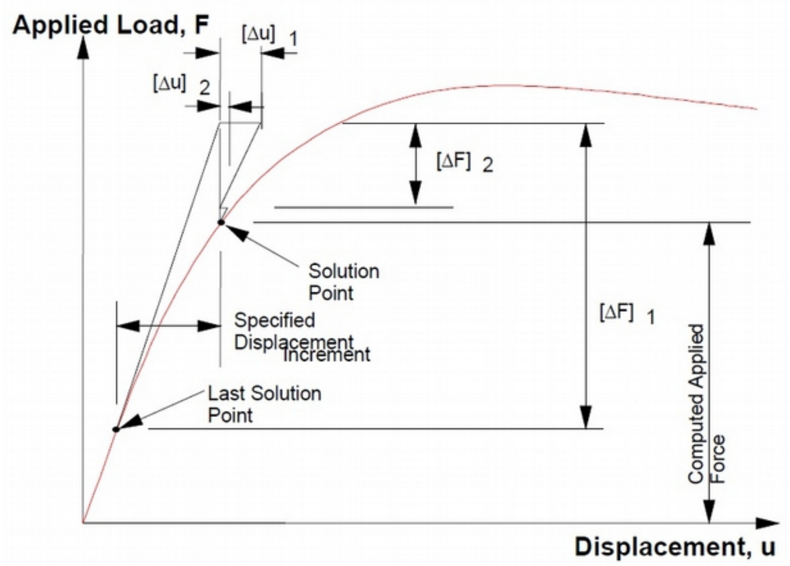


Fig.5. The Displacement Control Method (NIDA)

The Arc-Length Method

One of the most applicable techniques is the Arc-Length Method. In 1979, Riks introduced the constant arc-length which could pass the limit and turning points. Subsequently, Crisfield modified Riks' approach and established the cylindrical arc-length method. Afterwards, Fuji and Ramm (1981) investigated the path switching for bifurcation points in equilibrium paths.

The basic concept of the spherical arc-length method is to constrain the load increment so that the dot product of displacement along the iteration path remains a constant in the 2-dimensional plane of load versus deformation.

The main disadvantages of the original Arc-Length Method is that it is very complicated to implement, and the symmetrical nature of the tangent stiffness matrix is destroyed by the imposed additional constraint equation for displacement control. To overcome this problem, Crisfield (1981) and Ramm (1981) independently suggested an iterative process which separates the constraint equation from the set of equilibrium equation so as to retain the symmetrical and banded nature of the tangent stiffness matrix which is a common feature for finite element method of analysis.

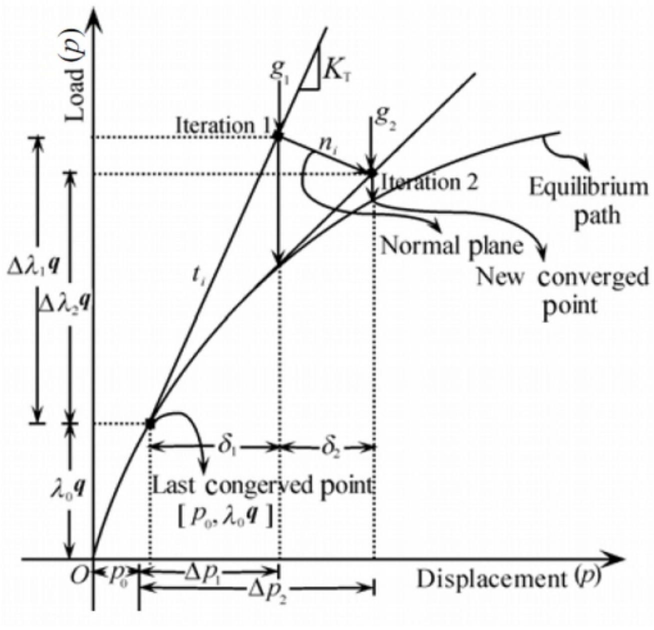


Fig.6. The Original Arc-Length Method (Riks, Wempner, 1971) using Constant Line Search

The Modified Arc-Length Method

Crisfield (1981) found that the Riks method was not suitable for standard finite element analysis even with modified Newton-Raphson (mN-R) procedure, because equations proposed by Riks destroy the banded nature of the stiffness matrix. For one-dimensional problem with N displacement variables, Crisfield (1981) gave the modification of the method and suggested the fixation of incremental length Δl during load increment using a special constraint. The proposed technique is termed as Cylindrical or Spherical Arc-Length Method or The Modified Arc-Length Method.

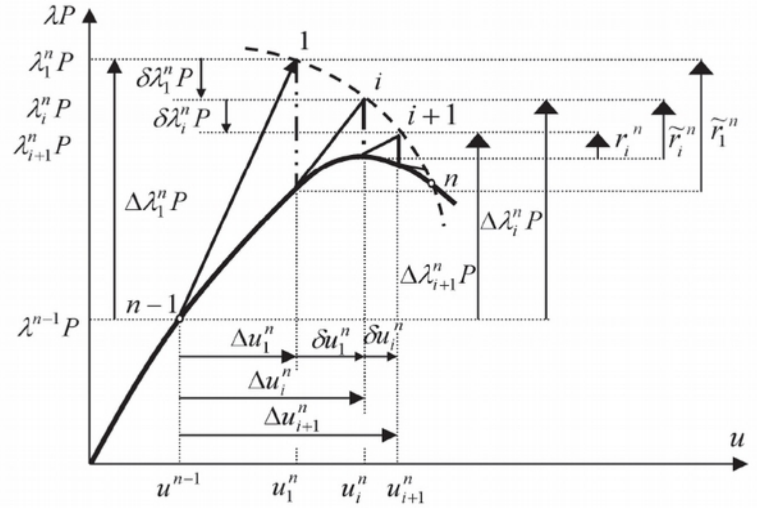


Fig.7. The Modified Arc-Length Method using Spherical Search and Accelerations (Crisfield, 1983)

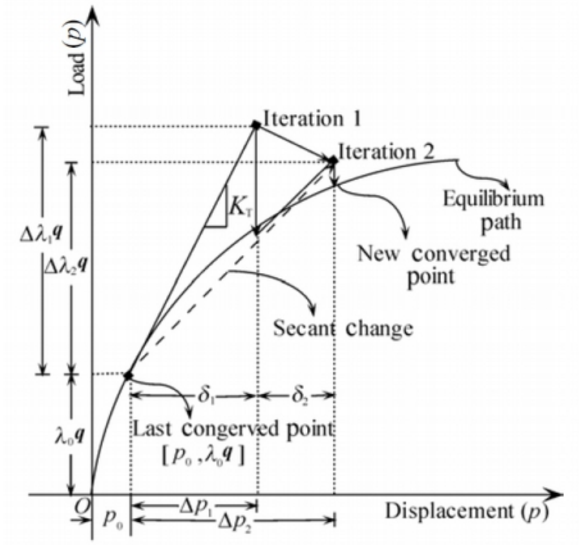


Fig.8. The Arc-Length Method with Secant Change (Ramm, 1981)

Due to its accuracy, reliability and satisfactory rate of convergence, The Modified Arc-Length Method is probably now the most popular method for nonlinear analysis, and it was noted to be robust and stable for pre- and post-buckling analysis.

The Minimum Residual Displacement Method

The basic idea of this method originally proposed by Chan (1988) is to minimize the norm of residual displacement in each iteration. The graphical representation of the procedure is demonstrated in Fig.9 showing the similarity with the Arc-Length Method.

From the Figure, it can be seen that this constraint condition enforces the iteration path to follow a path normal to the load-deformation curve. It adopts the shortest path to arrive at the solution path by error minimization and thus is considered to be an optimum solution. Due to its efficiency and effectiveness in tracing the equilibrium path, the minimum residual displacement method is usually chosen to perform the iterative procedure and combined with the part for load size determination in the first iteration by the arc-length method.

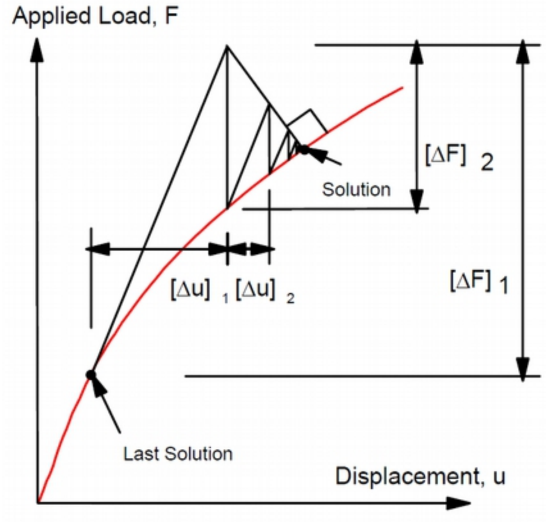


Fig.9. Minimum Residual Displacement Method (Memon, 2004)

Comparison of Existing Methods

The constant work, the arc-length and the minimum residual displacement methods are capable of tracing the nonlinear load-deformation curve with snap-through and snap-back phenomena. It has been generally observed that the minimum residual displacement method gives the most rapid rate of convergence and the highest reliability among these three methods. Better performance may be achieved when the minimum residual displacement iterative scheme is used in conjunction with the arc-length load increment for nonlinear static analysis.

Summary of difficulties of current method are :

a. Most methods are slow to converge near limit points and even fail to converge at all
b. The implementation of the most robust Arc-Length method requires long calculations
c. Load step is difficult to control and determine
d. Some methods can not trace the snap-through and snap-back behavior
e. Most methods can not use costly available information resulted from previous iterations (Previous Unfactored stiffness matrix, Previous Tangent stiffness matrix, Previous Load and Displacements Vectors)

A New Nonlinear Analysis Method using New Approaches

To provide a better nonlinear analysis method capable to deal with common engineering problems and to solve some difficulties found in existing methods, the author proposed a new nonlinear analysis method called **Direct Nonlinear Analysis using nathan-α Method** (or DNA nathan-α Method).

In all above analysis, only **one previous state** is used to find the next equilibrium state, while in this new method, **two previous states** will be used to find the next state.

In this method, a new approach utilizing previously calculated tangent stiffness matrix at previous state and equilibrium path at two previous states will be used to find the next equilibrium path.

Parallel to The Newton-Raphson approach, the new method is based on another very simple and well-known basic principle of Calculus, called **The Mean value Theory** as follows.

Mathematical Background

The **mean value theory** says : *For **any function** that is **continuous** on [a,b] and **differentiable** on (a,b) there **exists some c** in the interval (a,b) such that **the secant** joining the endpoints of the interval [a,b] is **parallel** to the **tangent at c**.*

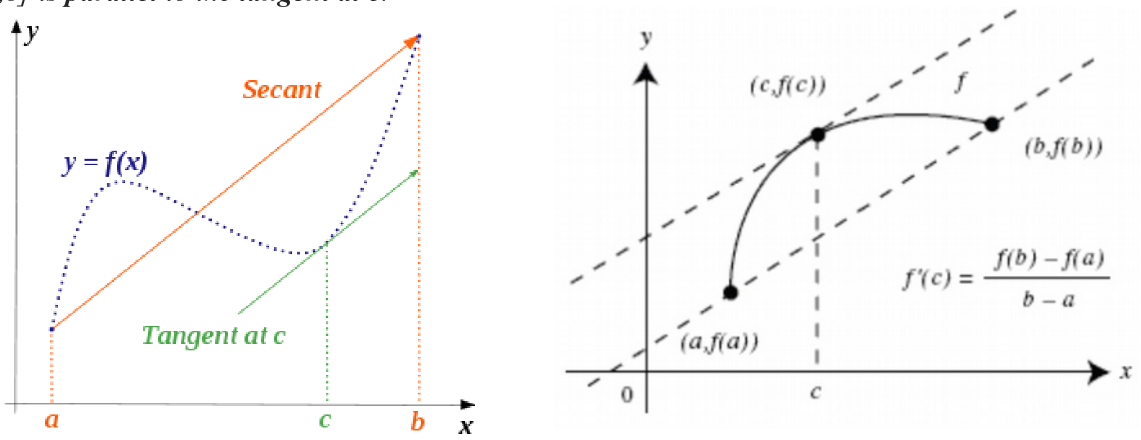


Fig.10a,b. Mean Value Theory (Wikipedia, 2015)

or in equation form:

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad (1)$$

In other words, for a segment of a curve from a to b, if the gradient of line from a to b is known, there will be at least one point c in the curve segment exists which has a tangent line equal to the gradient of the line. But from the theory, we can not say that for a certain point c with tangent m, there will always exist a line connecting a to b with gradient equal to m. This will create a problem in the proposed new method below.

Using a and c as two previously defined equilibrium states, and tangent at c as the tangent stiffness matrix of the structure, x as the displacement vector and y as the load vector, we will develop the direct nonlinear analysis procedure as follows.

Direct Nonlinear Analysis using nathan-α Method

We will apply above principles for solving nonlinear problems, to iterate along the equilibrium path. Using the fact that for almost any structures, the first region will be linear and the initial state will be well defined or zero, we can assume that at least two states F0 and F1 are ready to be used for calculating F2. Subsequently, F1 and F2 are available when calculating F3, and so on.

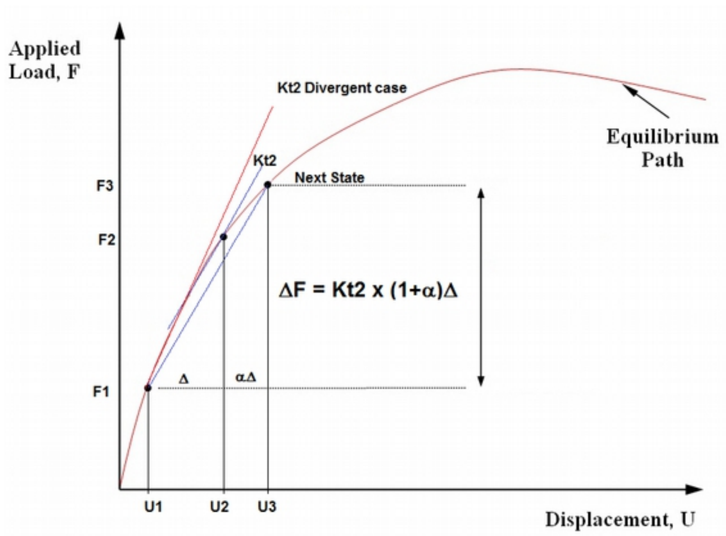


Fig. 11. Direct Nonlinear Analysis using α Method

If the curve of equilibrium path is not straight line, which is true for most nonlinear cases, using the above Mean Value Principle, there will be a point (u2,F2) having a tangent equal to the gradient of line connecting point 1 and point 3, where point 1 and 2 are already defined, but point 3 is still unknown. If we assume that point 3 is located $\alpha\Delta$ from point 2, we can calculate the load level at point 3 as:

$$F3 = F1 + Kt_2 \text{ x } (1+\alpha)\Delta \quad (2)$$
$$\Delta = (u2 - u1) \quad (3)$$

The value of α can be selected from the fact that correct value of α will give minimum residual forces between external nodal load vector and nodal load vector accumulated from element forces. It can be searched from simple procedure as follows:

1. Compute three residual forces F3i using 3 values of α : ($\alpha 1$,$\Delta F3_1$), ($\alpha 2$,$\Delta F3_2$), ($\alpha 3$,$\Delta F3_3$) where :

$$\Delta F3_1 = F3_{ext} - F3_{int} \quad (4)$$
$$\alpha 1 = 1.0 \quad \text{(good estimation for starting value)}$$
$$\alpha 2 = \alpha 1 - 0.1$$
$$\alpha 3 = \alpha 1 + 0.1$$

2. Using the three pair of values, solve the parabolic equation of residual forces:

$$\Delta F = a*\alpha^2 + b*\alpha + c \quad (5)$$

3. If the solution exists, there will be at least one root of equation (5) that will give $\Delta F = 0$, that is the next equilibrium state. We can use this value as α to calculate F3 using eq. 2. For most cases, because of the nature shape of the curve, there will be at least one real root for equation 5.

4. If the solution is not exist, that means the previous two states are almost in straight line, or still in linear region, or the tangent line of c does not intersect with the equilibrium path. In this case, the next position can be calculated using two alternatives:

   a. Using other third previous state that will not give a straight line condition so the quadratic equation will give valid root (recommended).

$$F3 = Fo + Kt_2 \text{ x } (1+\alpha)\Delta \quad (6)$$
$$\Delta = (u2 - u1) \quad (7)$$
$$\alpha 1 = 2.0 \text{ as good estimation for starting value}$$
$$\alpha 2 = \alpha 1 - 0.1$$
$$\alpha 3 = \alpha 1 + 0.1$$

   b. Using linear equation (assume that straight line will allow for linear interpolation):

$$F3 = F2 + Kt_2 \text{ x } \alpha\Delta \quad (8)$$

   In this case, we can use $\alpha = 1$ or smaller (0.5) to improve the accuracy of the results.

The above procedure is repeated from displacement value = 0 to u_{max}. The method above only uses unfactored tangent stiffness matrix (Kt_2) and simple calculations. So we do not need to find the costly and time consuming matrix inverse or factorization of Kt_2 that is not always available near bifurcation points. For each load step, mostly zero or 1 iteration is needed to reach the correct solution. Thus the name of the method is Direct Nonlinear Analysis nathan-α Method.
This means that the proposed method will be very stable, even near the limit points or passing buckling points. It is shown also that the stability of the solution is independent of the time step selected. The accuracy of the results depends only on selected value of Δ and procedure for computing internal forces of elements involved.

General Procedure

1. Initial stage (Linear region)

Known initial states: uo = 0, Fo=0, Kto (Tangent stiffness matrix at u=0, = Linear stiffness Ko)

Determine load step :

$$\Delta = Fmax/ndiv \quad (9)$$

where

ndiv = number of division required

Find : u1, F1 using Linear equation

$$u1 = [Kto]^{-1} \times F1 \quad (10)$$

Find $[Kt_i]$, do not need to factorize it

2. Next stage (Nonlinear region)

Using two equilibrium states (uo,Fo,Kto) dan (u1,F1) do Direct Nonlinear Analysis using nathan-α method described above.

Repeat until all points at equilibrium path have been traced.

Sample Problem

A simulation of well-known simple truss arch problem using cubic polynomial will be solved using the Direct Nonlinear Analysis using nathan-α method with various number of load steps. The Truss-Arch problem has a special feature called Snap-through, where if the load is still increasing, in certain point, the member force changed rapidly from compression state to tension state. The truss-arch will be unstable at snap-through region.

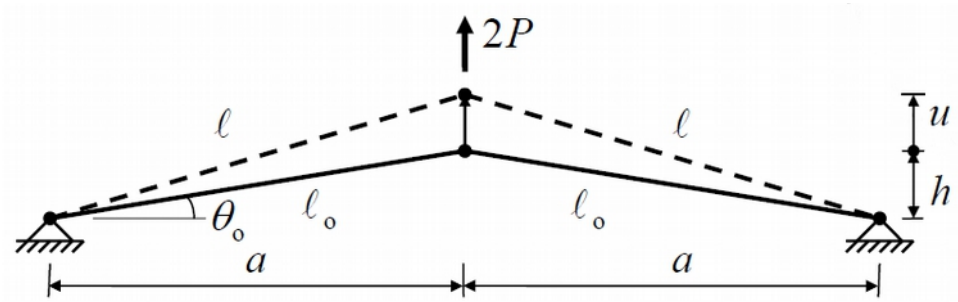


Fig.12. Truss Arch with Snap-through Problem (Mathisen, 2012)

The explicit nonlinear solution for the above Truss-Arch problems is:

$$\boxed{P = \frac{EA}{2\ell_o^3}\left[u^3 + 3hu^2 + 2h^2u\right] + N_o\left(\frac{u+h}{\ell_o}\right)} \quad \text{(Equation 11)}$$

Evaluating the result, it can be shown that the solution is stable, even for very few load steps, and will achieved high accuracy at minimum of 40 load steps. The correction procedure used in the method even can vary the load step depend on the curvature of the equilibrium path to get fewer load steps than the initial maximum load steps defined at the initial stage.

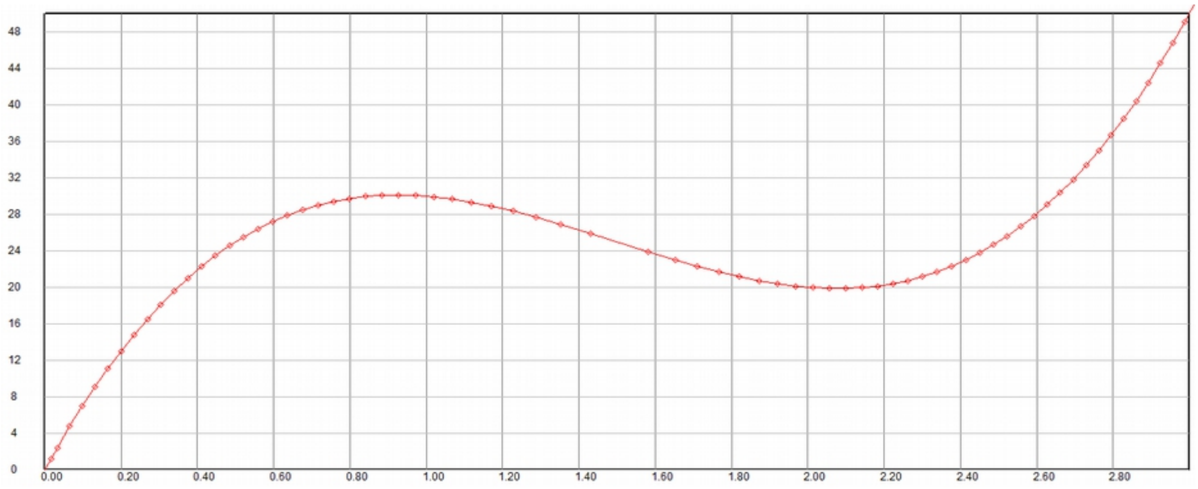


Fig. 13a. Simple truss, Number of load step=40 (red=theory, blue=this method)

The problem can be solved with high accuracy using only 40 load steps without iterations. It can be shown that the load or displacement step changes dynamically, smaller near softening region and limit points, and larger at flat region or hardening region. It can be seen that the blue line and the red line is coincided. From the analysis output, differences smaller than 1e-6 can be achieved without any iteration.

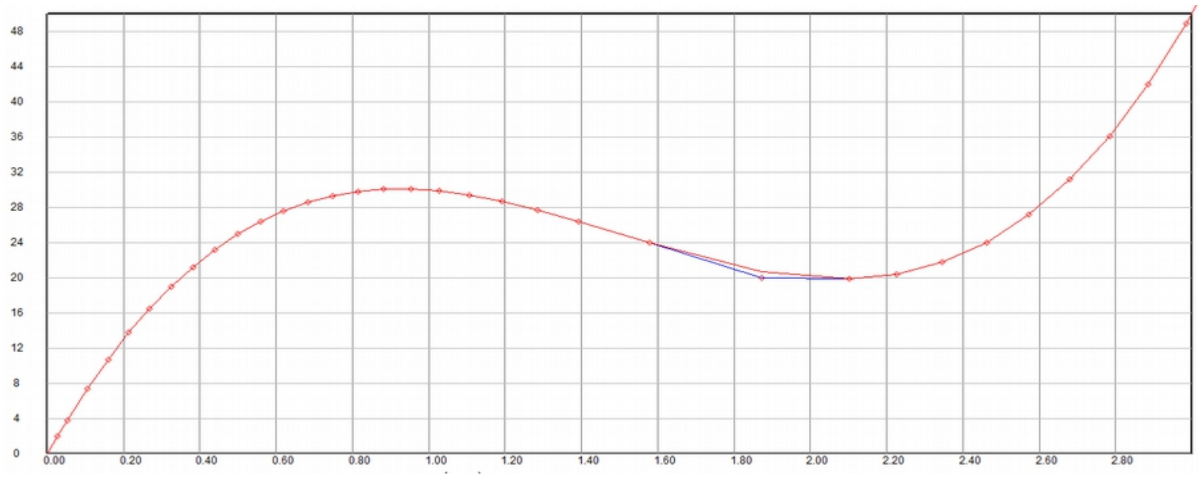


Fig. 13b. Simple truss, Number of load step=25

Even using fewer load steps (n=25), the proposed procedure (blue line) can trace correctly the first and other limit points and the hardening region. It diverge only a little bit near snap through region, but correct itself to the right path after that.

This auto-correction feature is one of many inherent properties of the method, where it will go back to the right path as long as the accuracy of the element forces calculation is quite good. This feature will give the **unconditionally stable** characteristic for the method. Without any special treatment, correction, or algorithms, the method can trace any point at equilibrium path correctly.

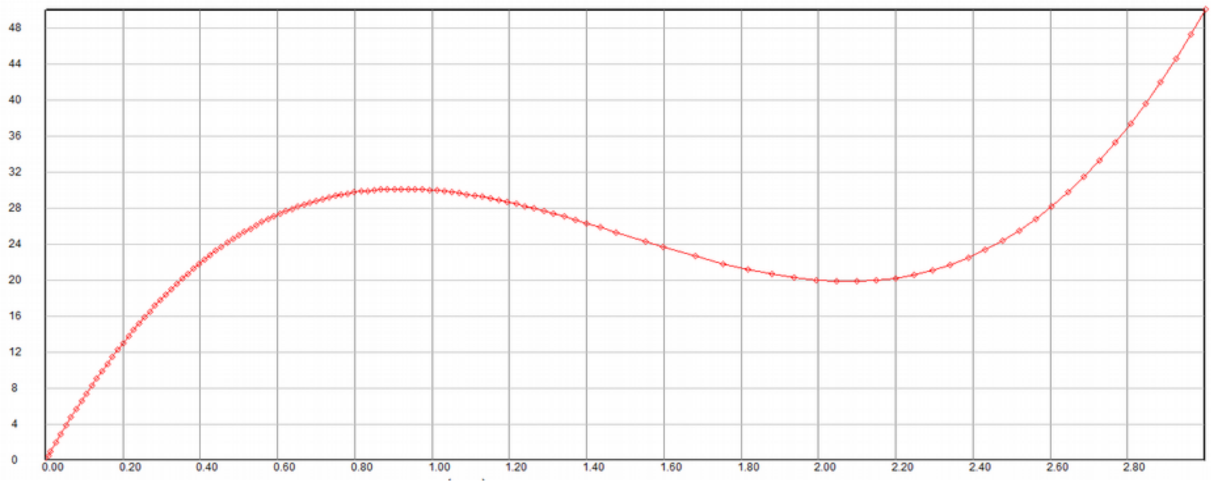


Fig. 13c. Simple truss, Number of load step=100

Using more load steps will give exactly the same result compared to number of load steps = 40. From this comparison, it can be shown that the proposed method is always stable and independent of the selected number of load steps. It can also pass any limit point, snap-through region, and hardening region without any difficulties. As inherent properties of the method, it can varies automatically the size of load step depends on the curvature of the equilibrium path, thus reducing the number of load steps accordingly.

Applications of Direct Nonlinear Analysis using nathan-α Method

Because the proposed method is straightforward, unconditionally stable, independent of number of load steps, does not need iteration, and has variable load step, it can be applied to unlimited number of fields: Steel Structures Analysis (Second Order Analysis, Buckling Analysis), Push Over Analysis, Soil-Structure Analysis, Slope Stability, Dome Structures with Snap-through effects, Geotechnical Problems, Tunneling, etc.

Due to its simple calculation procedure, the proposed method can be used to solve nonlinear problems graphically, or manually by hand calculation. It also can be implemented easily to any nonlinear computer programs.

Discussions

A new procedure for **Direct Nonlinear Analysis nathan-α Method** has been described above. The proposed procedure uses **new approaches** that have not been used in any nonlinear analysis methods before. It's unique approach to use Mean Value Theorem and using two previously defined equilibrium states instead of one, give it the long desired capability to **solve the nonlinear problem directly** without iterations, with **unconditionally stable condition**. These new approaches, although very simple, are neglected by others, during the intensive researches focusing on how to pass the limit points using only the previous state. The Mean Value Theorem is already available since the development of Calculus, but it has not been thought to be applicable in nonlinear analysis. So, the mathematicians are focusing on how to find the roots of nonlinear equation using Newton-Raphson scheme, while engineers are focusing on how to pass the limit points, which is the weakness of the Newton-raphson method. When we look back one equilibrium state, and apply the Mean Value Theorem, we came to the new, efficient and direct solution for the nonlinear problem.

The new procedure uses very simple Calculus principle of **Mean Value Theorem** to trace the next equilibrium state based on **two previously defined equilibrium states**. The **parameter α** can be solved directly using **quadratic interpolation**. From some cases, the initial value of α will be taken as 1.0 which so far has been proved to be the right choice for this method.

Using this method. it has been shown that **it is possible to solve incremental nonlinear equation directly**, almost without any iteration needed. The proposed method, naturally, by itself, can trace any point at equilibrium path, even the **softening region**, **limit points**, **snap-through** region, and **hardening** region without any difficulties. The method can also give **information about the type of region** at a certain equilibrium path (softening, post-buckling, hardening).

The proposed method is also **unconditionally stable** at limit points and does not need very small load step to get the correct equilibrium state. **The number of load steps** selected affects only the accuracy of the solution, and not the stability of the traced path. Inherently, the proposed method can **vary the size of the load steps** depend on the curvature of the equilibrium path.

In some rarely found cases, the straight line of the equilibrium path, the quadratic interpolation failed to give real roots for the parameter α. In this case, we can use previous equilibrium state and repeat the same procedure using larger value of $\alpha = 2.0$ and so on.

Conclusions

1. Using **Mean Value Theorem** and **two or three previously equilibrium states**, a nonlinear problems can be solved **directly** without iteration using **Direct Nonlinear Analysis nathan-α Method**

2. The proposed method shows **unconditionally stable** properties at any region along the equilibrium path, including **softening, hardening** and **snap-through**.

3. The proposed method can pass **limit points** and **bifurcation points** without any difficulties

4. The initial value of **α = 1.0** will give the best estimation to the correct solution

5. The proposed method can **vary dynamically the size of load steps** depends on the curvature of

6. The proposed method is **very effective and efficient** and can give **high accuracy** solution with only 40 load steps

7. The **correction procedure scheme** to use the third and the first previous equilibrium states will avoid the divergent solution

8. Because of **the use of two or three previously equilibrium states**, the proposed method needs to store at least **load and displacement vectors for 2 or 3 previous states**, and also need to store the **unfactored tangent stiffness of the previous state**.

Recommendations

1. The proposed method can be applied to efficiently solve nonlinear problems in many engineering fields.
2. The method must be implemented with correction scheme for invalid roots cases
3. To get correct result, the proposed method needs accurate tangent stiffness matrix formulation and accurate nodal forces calculation from element internal forces.
4. The method must be used with at least 40 load steps to get good results along the equilibrium path

Further Studies

1. The proposed method may be extended to Nonlinear Dynamics Problems
2. More accurate algorithms to determine the best initial value of α can be studied further
3. Although it is already very efficient, the proposed method can be combined with other existing methods, especially to solve special cases where the quadratic interpolation fails to give real roots.

References

Newton, Isaac, “De analysi per aequationes numero terminorum infinitas”, 1669, published 1736

Raphson, Joseph, “Analysis Aequationum Universalis”, London, 1690

Wempner, G. A., "Discrete Approximation Related to Nonlinear Theories of Solids," International Journal of Solids and Structures, Vol. 7, 1971, pp. 1581-1599.

Riks, E., "The Application of Newton's Methods to the Problem of Elastic Stability," Journal of Applied Mechanics, Vol. 39, 1972, pp. 1060-1066.

Riks, E., "An Incremental Approach to the Solution of Snapping and Buckling Problems," International Journal of Solids and Structures, Vol. 15, 1979, pp. 529-551.

Ramm, E., "Strategies for Tracing the Nonlinear Response Near Limit Point," Nonlinear Finite Element Analysis in Structural Mechanics, edited by W. Wunderlich, E. Stein, and K.-J. Bathe, Springer-Verlag, Berlin, 1981, pp. 63-89.

Crisfield, M. A., "A Fast Incremental/Iterative Solution Procedure That Handles Snap-Through," Computers & Structures, Vol. 13, 1981, pp. 55-62.

Crisfield, M. A., "An Arc Length Method Including Line Searches and Accelerations," International Journal for Numerical Methods in Engineering, Vol. 19, 1983, pp. 1269-1289.

Powell, G., and Simons, J., "Improved Iteration Strategy for Nonlinear Structures," International Journal for Numerical Methods in Engineering, Vol. 17, 1981, pp. 1455-1467.

Yang, Y. B., and McGuire, W., "A Work Control Method for Geometrically Nonlinear Analysis," NUMETA 85—1985 International Conference on Numerical Methods in Engineering: Theory and Applications, edited by J. Middleton and G. N. Pande, University College Swansea, Wales, UK, 1985, pp. 913-921.

Batoz, J-L., and Dhatt, G., "Incremental Displacement Algorithms for Nonlinear Problems," International Journal for Numerical Methods in Engineering, Vol. 14, 1979, pp. 1262-1267

MEMON Bashir-Ahmed#, SU Xiao-zu (苏小卒), “Arc-length technique for nonlinear finite element analysis”, Journal of Zhejiang University SCIENCE, ISSN 1009-3095, *J Zhejiang Univ SCI 2004 5(5):618-628*, 2004

NIDA, “Numerical Methods for Nonlinear Analysis”, Department of Civil & Environmental Engineering,The Hong Kong Polytechnic University, Hung Hum, Kowloon, Hong Kong, China, 2015

Kjell Magne Mathisen, “Solution Methods for Nonlinear Finite Element Analysis (NFEA)”, Lecture 11, Geilo Winter School, Norwegian University of Science and Technology, 2012

Hormoz Zareh,”Advanced Finite Element Analysis”, Portland State University, ME 565, Spring 2008, Portland, 2008

Weaver, William Jr, Gere, James, M., “Matrix Analysis of Framed Structures”, 2nd Edition, D. Van Nostrand Company, New York, 1980

Purcell, Edwin J., Varberg, Dale, “Calculus with Analytic Geometry”, 5th Edition, Prentice-Hall, 1987

Wikipedia, “Mean Value Theorem”, 2015

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2012	Development of Data Logger System for Consolidation Test, Lab Unpar, Bandung
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