“Effects of Horizontal and Vertical Earthquake Accelerations to Rooftop Pool during Medium Earthquake”

by Nathan Madutujuh (nathanmadutujuh@gmail.com)

**(Director of Engineering Software Research Centre,
Bandung, Indonesia)**

Summary

In this paper we will study the effects of horizontal and vertical earthquake acceleration on the water surface angle of Roof-top swimming pool on top of medium and tall buildings. In this study we will assume simplified condition of constant acceleration and neglect the effects of the shape of the pool. Simple formula for relation between water surface angle and earthquake acceleration will be derived and used for parametric study for Indonesian earthquake zone. To get the peak acceleration on roof top, a simplified equation derived from continuum system will be derived. The equation can be used to find the magnification factor that can be used to compute peak acceleration on roof top. From the parametric study, it is found that there is a great risk for water spill out for roof-top swimming pool, especially for medium rise building at medium risk earthquake zone. A simple design procedure is given following the discussion.

1. Introduction

Along with the increasing demand of high-rise apartments and hotels, the placement of pool at the medium height of building or rooftop of the building is become more popular. Although for few buildings, roof water tank can be used to to increase damping and reduce internal forces due to earthquake, for most of the buildings, it is not included in the analysis, because to get the actual response of the building needs special / advanced analysis, and roof- tank must be specially design for that purpose.

Fig.1. Roof Top Swimming Pool

From past earthquake experiences, it was found that the water of a pool can move out of the pool during moderate or strong earthquake. For example, during recent Nepal's earthquake, water can splash out of the pool easily, even for on the ground swimming pool. The effects will be greater for roof top swimming pool, especially the continuous type. Because the floor acceleration at top of building will be larger than the ground acceleration, a study is needed to find the effects of horizontal and vertical accelerations on water in rooftop swimming pool during earthquake.

Fig.2. Effect of Earthquake to a Swimming Pool on ground level
(water splash out the pool, from Youtube)

2. Relation between water surface angle and earthquake acceleration

In this paper we will conduct a simplified study about the effects. If we assume that earthquake will give constant acceleration, than a simple formula can be derived to relate the water surface angle and earthquake horizontal and vertical accelerations. The magnification effect caused by cyclic acceleration and effects due to pool shape will be neglected because including such parameters will require more advanced analysis using fluid dynamics theory.

g = gravity acceleration
ah = Horizontal acceleration in g
av = Vertical acceleration in g
rho = density of water
dv = volume of a water particle
dv = dA*dx
p = pressure of water = rho*(g-av)*h
h = depth of water from surface
h1 = depth of water at left of particle
h2 = depth of water at right of particle
h2 = h1 – dx*tan(theta)
theta = angle of water surface
dm = Particle mass = dV*rho

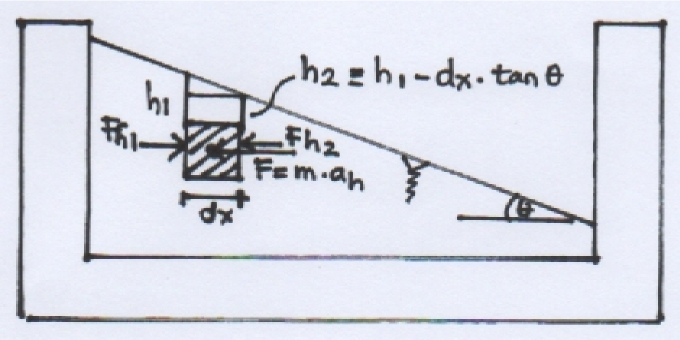


Fig.3. Free body of a water particle during earthquake

Equilibrium of Forces in Vertical Direction

Weight of particle = rho*dV*(g-av) (1)
Archimedes force = rho*dV*(g-av) (2)

Equilibrium of Forces in Horizontal Direction

dF = dm*a = dF1 – dF2 (3)

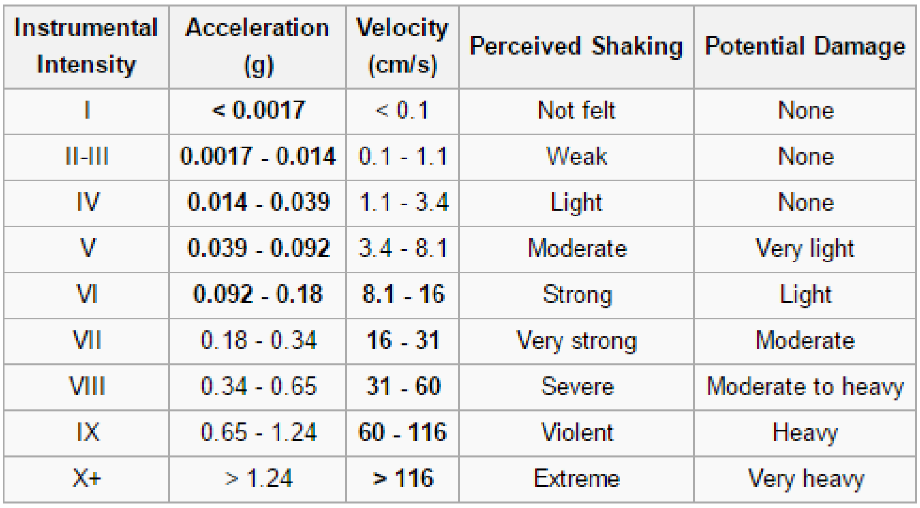
dm*a = rho*dA*dx*ah (3a)
dF1 = p1*dA = rho*(g-av)*h1*dA (3b)
h2 = h1 – dx*tan(theta) (3c)
dF2 = p2*dA = rho*(g-av)*h2*dA (3d)
dF2 = rho*(g-av)*(h1-dx*tan(theta))*dA (3e)
dF2 = rho*(g-av)*h1*dA – rho*(g-av)*dx*tan(theta)*dA (3f)

Substitute to : dF = dm*a = dF1 – dF2

rho*dA*dx*ah = rho*(g-av)*dx*tan(theta)*dA (4)
Tan(theta) = ah/(g-av) (5)
theta = angle of water surface due to earthquake acceleration

3. Estimated Peak Ground Acceleration

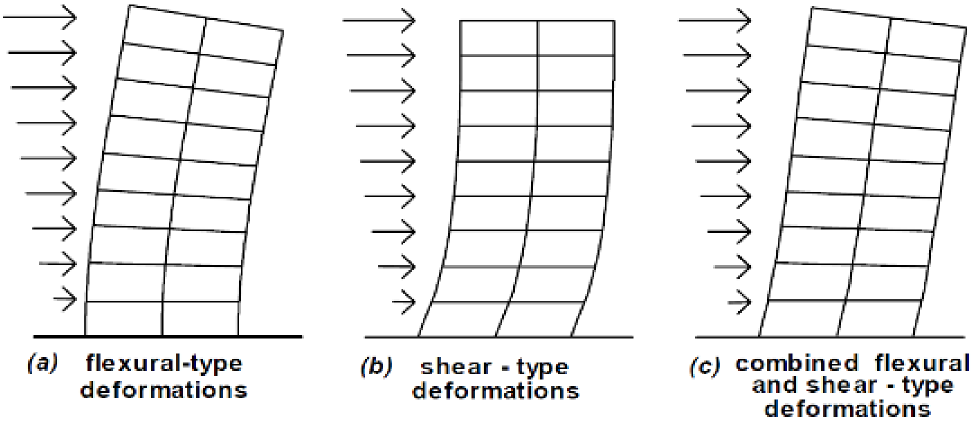
Table 1. Ground Acceleration (From USGS)



Instrumental Intensity	Acceleration (g)	Velocity (cm/s)	Perceived Shaking	Potential Damage
I	< 0.0017	< 0.1	Not felt	None
II-III	0.0017 - 0.014	0.1 - 1.1	Weak	None
IV	0.014 - 0.039	1.1 - 3.4	Light	None
V	0.039 - 0.092	3.4 - 8.1	Moderate	Very light
VI	0.092 - 0.18	8.1 - 16	Strong	Light
VII	0.18 - 0.34	16 - 31	Very strong	Moderate
VIII	0.34 - 0.65	31 - 60	Severe	Moderate to heavy
IX	0.65 - 1.24	60 - 116	Violent	Heavy
X+	> 1.24	> 116	Extreme	Very heavy

4. Peak Floor Acceleration at Top of Building

For a multistory building, floors above ground will experience greater horizontal acceleration than first floor nearer to ground by factor of PFA/PGA. This multiplier effect depends on the stiffness ratio and the height of the building. A study by Shahram TAGHAVI and Eduardo MIRANDA (1) give an estimation of Peak Floor Acceleration to Peak Ground Acceleration as follows:



(a) flexural-type deformations

(b) shear - type deformations

(c) combined flexural and shear - type deformations

Fig. 4. Lateral Deformations of Multistory buildings

Simplified Model

To find peak floor acceleration at roof top, one can use floor spectra analysis or more time consuming time history analysis. Because what we need is just the peak horizontal acceleration on roof top, we will use a simplified method using a simplified model of building as a continuum system.

If we assume that floor mass and story stiffness are constant through the height, we can derive a simplified continuum model for a multistory building deflects in bending and shear deformation as follows:

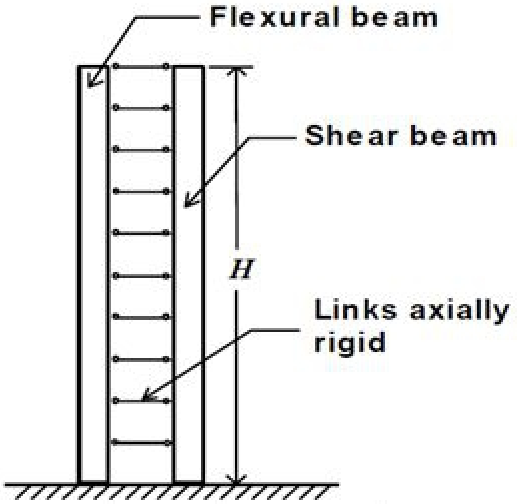


Fig.5. Simplified Model of Multistory Building (Miranda and Taghavi [2])

The governing dynamic equation of motion of the continuum system with uniform lateral stiffness shown in Figure 2 when subjected to a horizontal base acceleration of üg(t) is given by the following equation:

$$\frac{\rho}{EI_0}\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{c}{EI_0}\frac{\partial u(x,t)}{\partial t}+\frac{1}{H^4}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)-\frac{\alpha_0^2}{H^4}\frac{\partial}{\partial x}\left(\frac{\partial u(x,t)}{\partial x}\right)=-\frac{\rho}{EI_0}\frac{\partial^2 u_g(t)}{\partial t^2} \tag{6}$$

where ρ(x) is the mass per unit length in the model, u(x , t) is the lateral displacement at non-dimensional height x (varying between zero at the base of the building and one at roof level) at time t, H is the total height of the building, c(x) is the damping coefficient per unit length, Eio is the flexural rigidity at the base of the structure and α0 is the lateral stiffness ratio defined as:

$$\alpha_0 = H\left(\frac{GA_0}{EI_0}\right)^{1/2} \tag{6b}$$

For the case of uniform lateral stiffness, the dynamic characteristics can be obtained in closed form. In particular, the mode shape associated to the ith mode of vibration is given by (Miranda and Taghavi [2]):

$$\phi_i(x) = \frac{\sin(\gamma_i x) - \gamma_i\left(\alpha_0^2+\gamma_i^2\right)^{-1/2}\sinh\left(x\sqrt{\alpha_0^2+\gamma_i^2}\right) + \eta_i\left[\cosh\left(x\sqrt{\alpha_0^2+\gamma_i^2}\right) - \cos(\gamma_i x)\right]}{\sin(\gamma_i) - \gamma_i\left(\alpha_0^2+\gamma_i^2\right)^{-1/2}\sinh\left(\sqrt{\alpha_0^2+\gamma_i^2}\right) + \eta_i\left[\cosh\left(\sqrt{\alpha_0^2+\gamma_i^2}\right) - \cos(\gamma_i)\right]} \tag{7}$$

where η_i is defined as:

$$\eta_i = \frac{\gamma_i^2\sin(\gamma_i) + \gamma_i\sqrt{\alpha_0^2+\gamma_i^2}\,\sinh(\sqrt{\alpha_0^2+\gamma_i^2})}{\gamma_i^2\cos(\gamma_i) + \left(\alpha_0^2+\gamma_i^2\right)\cosh(\sqrt{\alpha_0^2+\gamma_i^2})} \tag{8}$$

Where Yi is an eigenvalue parameter associated with mode I and the root of the following characteristic equation:

$$2 + \left[2 + \frac{\alpha_0^4}{\gamma_i^2(\gamma_i^2+\alpha_0^2)}\right]\cos(\gamma_i)\cosh\left(\sqrt{\alpha_0^2+\gamma_i^2}\right) + \left[\frac{\alpha_0^2}{\gamma_i\sqrt{\alpha_0^2+\gamma_i^2}}\right]\sin(\gamma_i)\sinh\left(\sqrt{\alpha_0^2+\gamma_i^2}\right) = 0 \tag{9}$$

Once Yi is known for i-th mode of vibration, the modal participation factor and the period ratio of i-th mode are given by:

$$\Gamma_i = \frac{\int_0^1 \phi_i(x)dx}{\int_0^1 \phi_i^2(x)dx} \tag{10}$$

$$\frac{T_i}{T_1} = \frac{\gamma_1}{\gamma_i}\sqrt{\frac{\gamma_1^2+\alpha_0^2}{\gamma_i^2+\alpha_0^2}} \tag{11}$$

Examination of equations 3 to 7 shows that mode shapes, modal participation factors and period ratios are fully defined by a single parameter, the lateral stiffness ratio, α0. Miranda and Reyes [3] have indicated that this parameter can be estimated based on the type of lateral resisting system in the building. Shear wall and braced frame buildings usually have values of α0 between 0 and 1.5; building with dual structural systems consisting of a combination of moment-resisting frames and shear walls or a combination of moment-resisting frames and braced frames usually have values of α0 between 1.5 and 5; whereas moment-resisting frame buildings usually have values of α0 between 5 and 20

Using modal analysis, the absolute (total) floor acceleration at non-dimensional height x can be approximated as follows :

$$\ddot{u}^t(x,t) \cong \ddot{u}_g(t) + \sum_{i=1}^{N}\Gamma_i\,\phi_i(x)\ddot{D}_i(t) \tag{12}$$

where Di(t) is the relative acceleration of the i-th mode SDOF system subjected to ground acceleration. In above equation, modal participation factors and mode shapes are functions of lateral stiffness ratio, α0 and Di(t) is a function of period of the ith mode which is a function of α0 and T1, and the modal damping ratio ξ. So total acceleration at a certain location x can be computed by knowing fundamental period of vibration of the building T1, lateral stiffness ratio α0, modal damping ratio ξ and horizontal ground acceleration.

From above parametric study, we can have a simple relation between PFA and PGA for certain T1 and α0 as follows:

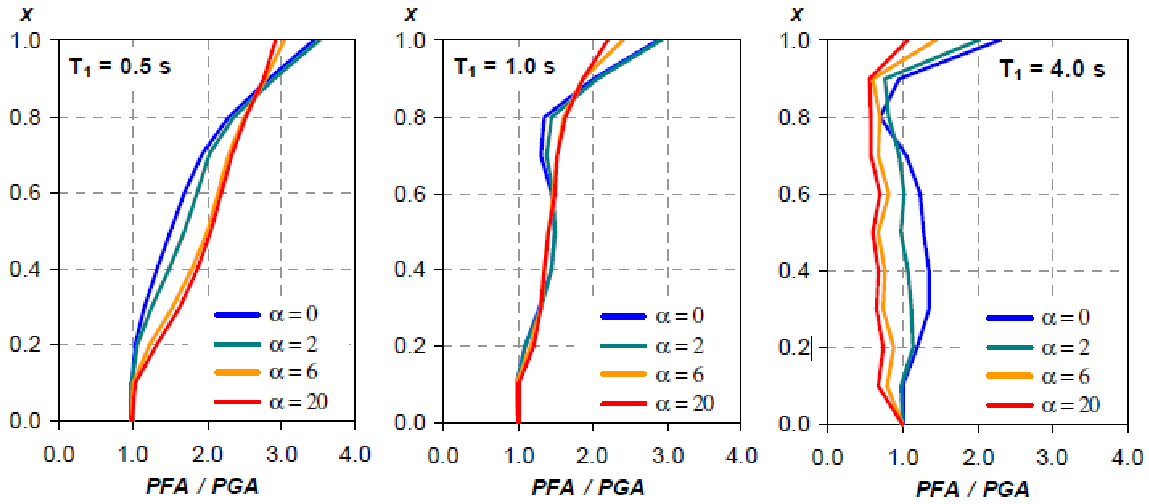


Fig.6. PFA/PGA for different values of T1 (Miranda and Taghavi [2])

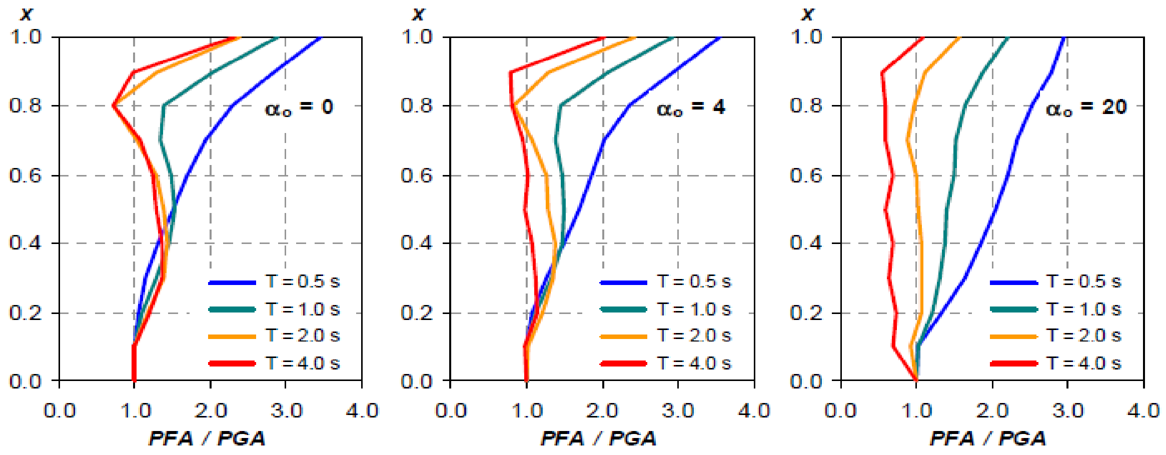


Fig.7. PFA/PGA for different values of α0 (Miranda and Taghavi [2])

Table 2. Estimated PFA/PGA Ratio: (summarized from Miranda and Taghavi, 2003)

Building System		No. of Floor	αo	T1	PFA/PGA
Moment Frame		5	5 – 20	0,5	3.0 – 3.5
		10	5 – 20	1,0	2.2 – 2.4
Shearwall		20	0 – 1.5	1,0	2,1
Braced Frame		20	0 – 1.5	1,0	2,1
Dual System		20	1.5 – 5.0	2,0	2,1
Dual System		40	1.5 – 5.0	4,0	2,0

From above study, it is found that the estimated ratio of PFA/PGA is > 2.0. Based on estimated ground acceleration for Scale >= VI (0.2 – 0.3 g) and PFA/PGA ratio (2.0), a parametric study has been done on the effects of earthquake acceleration on water in roof top swimming pool as follows:

5. Parametric Study

In this parametric study, three swimming pool sizes (Small, Medium, Large) have been considered with depth of 1.2 – 1.5m. Horizontal ground acceleration of 0.1-0.3 and vertical ground acceleration of 0.1-0.2 also considered.

Earthquake acceleration will be assumed constant, pool shape effects and resonance effect are neglected. Safety Status will be checked for the angle of water surface is larger than the maximum angle allowed by pool size, in that case, the water will be moved from the pool during the earthquake. Safety Factor used for this calculation is 1.0.

Table 3: SMALL POOL

Width of Pool = W	=	4	m	4	4	4	4
Depth of Pool = D	=	1,2	m	1,2	1,2	1,2	1,2
Horizontal acceleration, ah	=	0,1		0,15	0,2	0,25	**0,3**
Vertical Acceleration, av	=	0,1		0,1	0,1	0,1	**0,1**
PFA/PGA	=	2		2	2	2	2
Peak Acceleration, ah,max		**0,2**		**0,3**	**0,4**	**0,5**	**0,6**
Tan(theta)	=	0,2222		0,3333	0,4444	0,5556	0,6667
theta	=	12,5288	degrees	18,4349	23,9625	29,0546	33,6901
Tan(theta),max = D/W	=	0,3		0,3	0,3	0,3	0,3
theta, max	=	16,6992	degrees	16,6992	16,6992	16,6992	16,6992
STATUS	=	**OK**		**NOT OK**	**NOT OK**	**NOT OK**	**NOT OK**

Table 4: MEDIUM POOL

Width of Pool = W	=	6	m	6	6	6	6
Depth of Pool = D	=	1,2	m	1,2	1,2	1,2	1,2
Horizontal acceleration, ah	=	0,1		0,15	**0,2**	**0,25**	**0,3**
Vertical Acceleration, av	=	0,1		0,1	**0,1**	**0,1**	**0,1**
PFA/PGA	=	2		2	2	2	2
Peak Acceleration, ah,max		**0,2**		**0,3**	**0,4**	**0,5**	**0,6**
Tan(theta)	=	0,2222		0,3333	0,4444	0,5556	0,6667
theta	=	12,5288	degrees	18,4349	23,9625	29,0546	33,6901
Tan(theta),max = D/W	=	0,2		0,2	0,2	0,2	0,2
theta, max	=	11,3099	degrees	11,3099	11,3099	11,3099	11,3099
STATUS	=	**NOT OK**		**NOT OK**	**NOT OK**	**NOT OK**	**NOT OK**

Table 5: LARGE POOL

Width of Pool = W	=	8	m	8	8	8	8
Depth of Pool = D	=	1,5	m	1,5	1,5	1,5	1,5
Horizontal acceleration, ah	=	0,1		0,15	**0,2**	**0,25**	**0,3**
Vertical Acceleration, av	=	0,1		0,1	**0,1**	**0,1**	**0,1**
PFA/PGA	=	2		2	2	2	2
Peak Acceleration, ah,max		**0,2**		**0,3**	**0,4**	**0,5**	**0,6**
Tan(theta)	=	0,2222		0,3333	0,4444	0,5556	0,6667
theta	=	12,5288	degrees	18,4349	23,9625	29,0546	33,6901
Tan(theta),max = D/W	=	0,1875		0,1875	0,1875	0,1875	0,1875
theta, max	=	10,6197	degrees	10,6197	10,6197	10,6197	10,6197
STATUS	=	**NOT OK**		**NOT OK**	**NOT OK**	**NOT OK**	**NOT OK**

6. Conclusions

1. Peak Floor Acceleration is usually larger than twice of Peak Ground acceleration
2. Pool with smaller Depth to Width ratio will have higher risk of water movement during medium earthquake
3. Vertical acceleration will increase water movement during medium earthquake
4. Higher risk of water movement for rooftop swimming pool during small and medium earthquake if ah > 0.15g
5. More higher risk for human body swimming on surface of water during earthquake
6. Highest risk is found at buildings with number of floor <= 5 and almost constant after that
7. Resonance effects must be included if the time period of water and time period of building coincided.

7. Recommendations

1. Rooftop swimming pool must be designed to prevent casualties caused by water movement during earthquake
2. Further study needed for effects of earthquake on the human body inside a rooftop pool during earthquake
3. Special study will be needed for swimming pool with irregular shape using 3D Fluid Dynamics Software
4. For rooftop pool with NOT OK status, a safety wall with enough height must be added around the pool
5. It is not recommended to use rooftop pool for medium and high-rise buildings at highly seismic area

8. Recommended design procedure is as follows

1. Known : D (water depth), W (pool width), ah, av (earthquake horizontal and vertical accelerations), NF (number of floors)
2. Find estimated building's Time Period To , modal damping ratio ξ, lateral stiffness ratio α0
3. Using Table 2, with To and NF, find the PFA/PGA factor
4. Using PFA/PGA factor, find peak acceleration at roof-top, ahp
5. Vertical acceleration will be assumed same as on the ground, av
   Using equation (5), find the estimated water surface angle
6. Compute max angle using Tan(theta), max = D/W
7. Check for safety status of the pool, and increase the side wall of the pool to anticipate the water movement

9. Bibliography

1. Shahram TAGHAVI and Eduardo MIRANDA, “Estimation of seismic acceleration demands in building”, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, August 1-6, 2004, Paper No. 3199

2. Miranda, E.; and Taghavi. S. (2003), “Approximate floor acceleration demand in multi- tory buildings”, Part I: Theory”, Submitted for review and possible publication to Journal of Structural Engineering

3. Miranda, E.; and Reyes, C.J. (2002), "Approximate lateral drift demands in multi-story buildings with non-uniform stiffness,” J. Struct. Engrg., ASCE, 128(7), 840-849.

4. David Halliday and Robert Resnick, “Physics”, 3rd Edition, Jonh Wiley and sons, 1978